Azonosító jel:

ÉRETTSÉGI VIZSGA • 2010. október 19.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2010. október 19. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is over, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four out of the five problems**. When you have finished the examination paper, write in the square below the number of the problem NOT selected.**
   If it is not clear for the examiner which problem you do not want to be assessed, then problem 9 will not be assessed.

4. In solving the problems, you are allowed to use a calculator that cannot store and display textual information. You are also allowed to use any book of four-digit data tables. The use of any other electronic devices, or printed or written material is forbidden!

5. **Always write down the reasoning used in obtaining the answers, since a large part of the attainable points will be awarded for that.**

6. **Make sure that the calculations of intermediate results can also be followed.**

7. In solving the problems, theorems studied and given a name in class (e.g. the Pythagorean theorem or the altitude theorem) do not need to be stated precisely. It is enough to refer to them by the name, but their applicability needs to be briefly explained. Reference to other theorems will only be awarded full mark if the theorem and all its conditions are stated correctly (proof is not required), and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too.

9. Write in pen. The examiner is instructed not to mark anything in pencil, other than diagrams. Diagrams are also allowed to be drawn in pencil. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In the case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

11. Please do not write anything in the grey rectangles.

I.

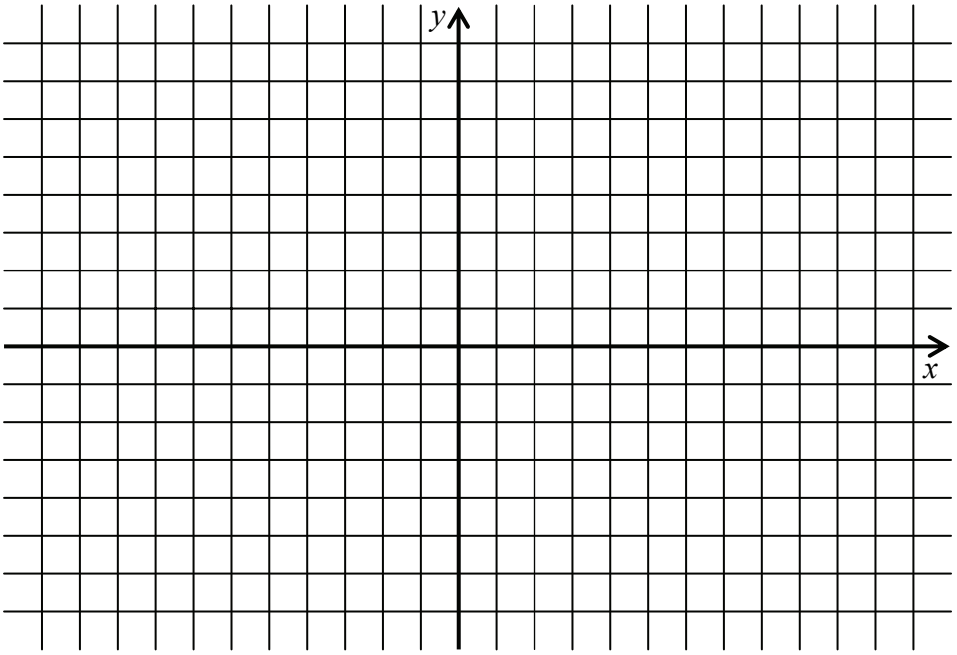
1.

a) Find all real numbers satisfying the inequality below.
$(x-1)^3-(x+1)^3>-8$

b) The functions f and g are both defined on the interval $[-3;6]$.
$f(x)=\sqrt{x+3}$ and $g(x)=-0.5x+2.5$.
On the same set of coordinate axes, graph the functions f and g on the interval $[-3;6]$.
Prove by calculation that the coordinates of the point of intersection of the two graphs are both integers.

c) Solve the inequality below on the set of real numbers.
$0.5x+\sqrt{x+3}\leq 2.5$

a)	4 points	
b)	4 points	
c)	6 points	
T.:	14 points	


y
x

2.

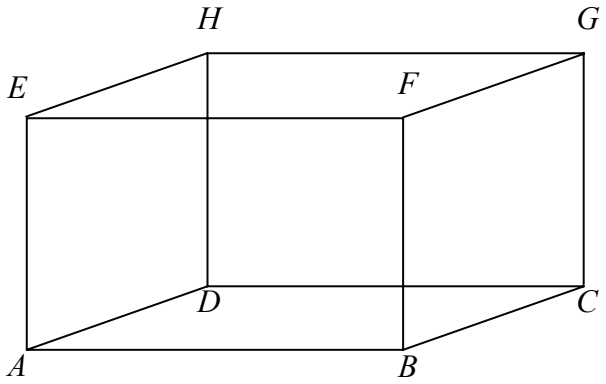
a) How many ten-digit positive integers are there in which all digits are elements of the set {0 ; 8}?

b) Find the smallest positive multiple of 45 that only contains digits of 0 and 8.

(In solving this problem, you are advised to consider instructions 5 and 6 on page 3 very carefully.)

a)	3 points	
b)	7 points	
T.:	10 points	

3. The edges from vertex *A* of the cuboid *ABCDEFGH* are *AB=12; AD=6; AE*=8. Let *P* denote the midpoint of edge *HG*.



a) Calculate the surface area of pyramid *ABCDP*.

b) Find the measure of the angle enclosed by the planes of the faces *ABP* and *ABCD* of the pyramid *ABCDP*.

a)	10 points	
b)	3 points	
T.:	13 points	

4. In a survey, 640 families were asked how many children there were in the family and of what sex. The results of the survey are tabulated below.

		number of boys					
		0	**1**	**2**	**3**	**4**	**5**
number of girls	**0**	160	103	61	8	5	0
	1	121	58	11	4	1	1
	2	54	15	3	2	2	2
	3	9	3	1	1	0	1
	4	6	3	1	1	1	0
	5	1	0	1	0	0	0

(That is, for example there were 160 families in the sample with no children, and 15 families with 1 boy and 2 girls.)

a) How many boys are there altogether in the families taking part in the survey?

b) In the sample, what is the most frequently occurring number of girls in families with at least two children?

c) Those families in the survey with at least four children are also provided financial support by a community aid organization. Complete the table below to obtain a frequency table for the number of children in the supported families.

number of children in a family	4	5	6	7	8	9	10
frequency							

How many families are supported by the community aid service, and how many children does that involve altogether?

a)	3 points	
b)	5 points	
c)	6 points	
T.:	14 points	

II.

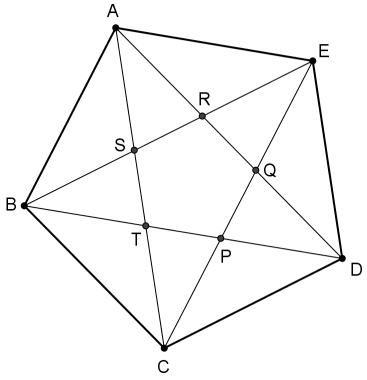
You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

5. The parabola $x^2 = 2y$ cuts the circular disc of equation $x^2 + y^2 \leq 8$ into two parts. Find the area of the convex part. Leave π in your answer, do not use an approximate value in your calculations.

T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

6. The diagram shows a regular pentagon *ABCDE* with all its diagonals drawn. The intersections of the diagonals are *P, Q, R, S, T*, as labelled in the diagram.



a) How many triangles are there in the diagram whose vertices are all among the 10 points marked, and in which all sides lie along the lines of the sides and diagonals of the pentagon *ABCDE* ?

How many significantly different triangles are there in this set of triangles if similar triangles are not considered significantly different?

b) Given that the area of quadrilateral *ABCQ* is 120 cm^2, find the area of pentagon *ABCDE*.
Round your answer to the nearest integer.

c) The given diagram can be considered a graph of ten vertices. The statements below refer to this graph. Decide and briefly explain whether each statement is true or false.

Statement 1.: The graph has 20 edges.
Statement 2.: The graph contains a subgraph that is a circuit of eight edges.

a)	8 points	
b)	4 points	
c)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

7. A cosmetics company manufactures a certain kind of cream in large quantities. The total monthly production (x kilograms) is between 100 kg and 700 kg. By contract, it is all sold to a wholesale merchant in the same month. The contract also states that the merchant is to pay a price of $(36–0.03x)$ euros per kilogram for the cream.
The monthly production costs (expenses) also depend on the quantity of cream manufactured. The total monthly cost related to cream production, is given by the cost function $0.0001x^3 - 30.12x + 13000$, also in euros.

a) Calculate the quantity of cream in kilograms for which the total monthly revenue (income) is a maximum. What is the maximum monthly income?

b) What is the maximum monthly profit that the company can make by manufacturing cream? For what quantity of cream (in kilograms) will that profit be achieved? (*profit = income – cost*)

a)	6 points	
b)	10 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

8.

a) Two boys, Miki and Karcsi each bought scratch tickets for 240 forints. Each of them paid in coins, and they paid the exact amount. (There are 5, 10, 20, 50, 100 and 200-forint coins.) Miki paid with 4 coins and Karcsi paid with 5 coins. In how many different ways may each of them have paid?
(The order of handing over the coins does not count.)

In Gamblia, there are two lottery draws a day. Bandi plays a single ticket. With the **same ticket**, he may win in **both draws** on that day.

b) In every draw, the probability of a certain ticket to win the jackpot is p. $(0 < p < 1)$ What is the probability that on a particular day Bandi will win the jackpot at least once?

There is a change in the rules: there is **only one draw** per day (with every other rule remaining the same). Now Bandi plays two tickets (not necessarily filled out differently).

c) What is the probability that on a particular day Bandi will have a jackpot winner ticket?

d) If Bandi wants to win the jackpot, which construction is more favourable for him: the game described in **b)** or the game described in **c)**?

a)	4 points	
b)	4 points	
c)	4 points	
d)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

9. Statistics based on the educational records of 10 580 students of a university reveal the following information about English and German language certificates:
Out of those not having a certificate in German, 70% have no certificate in English, and out of those with a certificate in German, 30% have no certificate in English.
60% of those not having a certificate in English do not have a certificate in German either.

a) How many students have a certificate in English, and how many students have a certificate in German?

b) What percentage of the students have certificates in both English and German?

a)	12 points	
b)	4 points	
T.:	16 points	

	number of problem	maximum score	points awarded	maximum score	points awarded
Part I	1.	14		**51**	
	2.	10			
	3.	13			
	4.	14			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date examiner

	score awarded, rounded to **integer** (elért pontszám **egész számra** kerekítve)	**integer** score entered in program (programba beírt **egész** pontszám)
Part I (I. rész)		
Part II (II. rész)		

examiner (javító tanár) registrar (jegyző)

date (dátum) date (dátum)